6.6 Pascal's Triangle

A collection of numbers laid out in rows and diagonals that have many unique traits

Blaise Pascal

Row — D

Row 0
Diagonal 0

R_0 1 D_0
R_1 1 1 D_1
R_2 1 2 1 D_2
R_3 1 3 3 1 D_3
R_4 1 4 6 4 1
R_5 1 5 10 10 5 1
R_6 1 6 15 20 15 6 1

$_4C_4 = 1$ how many ways can you choose all of them

$_4C_0 = 1$ how many ways can you choose none of them

$_4C_1 = 4$ ABCD
ABC, ABD, ACD, BCD

$_4C_3 = 4$

$_4C_2 = 6$ AB AC AD
BC BD
CD

Direct Correlation to Combinations

Pascals Δ
$_nC_r$
n = row number
r = diagonal

$_0C_0$
$_1C_0$ $_1C_1$
$_2C_0$ $_2C_1$ $_2C_2$
$_3C_0$ $_3C_1$ $_3C_2$ $_3C_3$

$_5C_3$

$$_5C_3 = \frac{5!}{(5-3)!3!}$$
$$= \frac{5 \cdot 4 \cdot \cancel{3 \cdot 2 \cdot 1}}{(2 \cdot 1)(\cancel{3 \cdot 2 \cdot 1})}$$
$$= 10$$

another useful thing

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$r_0 = 1 = 2^0$
$r_1 = 2 = 2 = 2^1$
$r_2 = 4 = 2 \cdot 2 = 2^2$
$r_3 = 8 = 2 \cdot 2 \cdot 2 = 2^3$
$r_4 = 16 = 2 \cdot 2 \cdot 2 \cdot 2 = 2^4$
$r_5 = 32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$
$r_6 = 64 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6$

sum of $r_n = 2^n$

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

flip a coin 5 times

number of results $r_5 = 2^5 = 32$

0 1 2 3 4 5 ⇒ Diagonals
1 5 10 10 5 1 Number of Heads

How many results have exactly 0 heads? 1

more than 3 6

less than 3 16