6.2   Counting and Permutations

What did we notice yesterday

2 apps   3 entrees   2 desserts
2 × 3 × 2 = 12
6 sides   6 sides
6 × 6 = 36

Multiplication Counting
Principle
To find the total number
of possible ways, multiply
at each stage the number of options
2 × 3 × 2 = 12
7·6·5·4·3·2·1 = 5040

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2 dice   8 $\underset{\text{spinner}}{\text{section}}$ 4 section spinner

36 · 8 · 4

1152

Factorial (!)
a short hand notation
for multiplying consecutive
numbers
$2! \rightarrow 2·1 = 2$
$3! \rightarrow 3·2·1 = 6$
$7! \rightarrow 7·6·5·4·3·2·1 = 5040$
$5! \rightarrow 5·4·3·2·1 = 120$

Two special factorials
$1! \rightarrow 1 = 1$
$0!$ special case whose
value is 1

We use to count single
items in many situations

How many ways can you arrange
ABCD

ABCD BACD CABD DABC
ABDC BADC
ACBD
ACDB   24 ways
ADBC $4 \cdot 3 \cdot 2 \cdot 1 = 24$
ADCB $4!$

If order is important

$ABC \neq BAC$

We use a process called Permutations

Symbolically n = number of options

$_nP_r$ r = number we use

$_4P_4 = 24$

formula $_nP_r = \dfrac{n!}{(n-r)!}$

$_4P_4 = \dfrac{4!}{(4-4)!} = \dfrac{4 \cdot 3 \cdot 2 \cdot 1}{4} = 24$

$_4P_3 = \dfrac{4!}{(4-3)!} = \dfrac{4 \cdot 3 \cdot 2 \cdot 1}{1} = 24$

$_4P_1 = \dfrac{4!}{(4-1)!} = \dfrac{4!}{3!} = \dfrac{4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 4$

SPREAD $_6P_4 = \dfrac{6!}{(6-4)!} = \dfrac{6!}{2!}$

$6 = n$
$4 = r$

$= \dfrac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 360$

7 songs choosing 5

$_7P_5 = \dfrac{7!}{(7-5)!} = \dfrac{7!}{2!} = \dfrac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1}$

$= 2520$