# 3.7 Matrix Multiplication

It is not addition or subtraction nor is it scalar

Dimensions are important

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Dimensions

$A = 3 \times 2$

$B = 2 \times 4$

$A \cdot B \quad 3 \times \boxed{2 \cdot 2} \times 4 = 3 \times 4$

We want the column in $M_1$ to be the same as the row in $M_2$

$R \times C$

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$1 \times 5 \cdot 5 \times 1 = 1 \times 1$

$2 \times 2 \cdot 2 \times 4 = 2 \times 4$

$5 \times 1 \cdot 1 \times 5 = 5 \times 5$

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When multiplying we take a row and multiply by a column

$$\begin{bmatrix} 5 & 4 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \\ 5 \end{bmatrix}$$

$(5 \times 2) + (4 \times 6) + (-2 \times 5) = 24$

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$$\begin{array}{cc} & \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{array}{c} R_1 \\ R_2 \end{array} & \begin{bmatrix} R_1 & R_1 \\ R_2 & R_2 \end{bmatrix} \end{array} \cdot \begin{array}{cc} & \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{array}{c} R_1 \\ R_2 \end{array} & \begin{bmatrix} C_1 & C_2 \\ C_1 & C_2 \end{bmatrix} \end{array} = \begin{bmatrix} R_1C_1 & R_1C_2 \\ R_2C_1 & R_2C_2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 6+6 & 12+2 \\ 2+15 & 4+5 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 14 \\ 17 & 9 \end{bmatrix}$$

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$3 \times \boxed{2 \times 2} \times 4 = 3 \times 4$

$$\begin{bmatrix} 2 & 1 \\ 4 & 3 \\ -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & -2 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2+5 & 4-2 & 6+0 & 8-1 \\ 4+15 & 8-6 & 12+0 & 16-3 \\ -1+25 & -2-10 & -3+0 & -4-5 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 2 & 6 & 7 \\ 19 & 2 & 12 & 13 \\ 24 & -12 & -3 & -9 \end{bmatrix}$$

Look into future

Identity

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$x - 5$

$y - 2$

$$\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -5 \\ -2 \end{bmatrix}$$

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## 3.7 Matrix Multiplication

Is not addition/subtraction or scalar

Dimensions are important

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Dimensions - The r x c of $M_1$ and the r x c of $M_2$ determine if we can multiply and the dimensions of the product

They must have a common interior size (c of $M_1$ = R of $M_2$)

$$2 \times \boxed{3 \cdot 3} \times 4 = 2 \times 4$$

$$A \cdot B =$$

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$$1 \times 5 \cdot 5 \times 1 = 1 \times 1$$

$$2 \times 3 \cdot 3 \times 2 = 2 \times 2$$

$$5 \times 1 \cdot 1 \times 5 = 5 \times 5$$

$$3 \times 2 \cdot 2 \times 3 = 3 \times 3$$

commutative
switch order

$$A \cdot B \neq B \cdot A$$

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To multiply we look at

$R_1 \longrightarrow C_2$

$$\begin{bmatrix} \cdot & - & - \\ \cdot & - & - \\ a & b & c \\ - & - & \cdot \end{bmatrix} \begin{bmatrix} \vdots & \vdots & d & \vdots \\ \vdots & \vdots & e & \vdots \\ \vdots & \vdots & f & \vdots \end{bmatrix}$$

$ad + be + cf$

$$\begin{array}{l} R_1 \\ R_2 \end{array}\begin{bmatrix} \text{===} \\ \text{===} \end{bmatrix} \begin{array}{cc} C_1 & C_2 \\ \begin{bmatrix} \| & \| \end{bmatrix} \end{array} = \begin{bmatrix} R_1C_1 & R_1C_2 \\ R_2C_1 & R_2C_2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3\cdot 2+2\cdot 3 & 3\cdot 4+2\cdot 1 \\ 1\cdot 2+5\cdot 3 & 1\cdot 4+5\cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 14 \\ 17 & 9 \end{bmatrix}$$

$A = \begin{bmatrix} 1 & 3 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$

$AB$ $\begin{bmatrix} 1 & 3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1\cdot 4 + 3\cdot 2 + 0\cdot -1 \end{bmatrix} = \begin{bmatrix} 10 \end{bmatrix}$

$BA$ $\begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 12 & 0 \\ 2 & 6 & 0 \\ -1 & -3 & 0 \end{bmatrix}$

$3\times 2 \cdot 2\times 4$ $\quad 3\times 4$

$$\begin{bmatrix} 2 & 1 \\ 4 & 3 \\ -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & -2 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2+5 & 4-2 & 6+0 & 8-1 \\ 4+15 & 8-6 & 12+0 & 16-3 \\ -1+25 & -2-10 & -3+0 & -4-5 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 2 & 6 & 7 \\ 19 & 2 & 12 & 13 \\ 24 & -12 & -3 & -9 \end{bmatrix}$$