2.6 Proving Statements about Angles

Standards/Objectives

Standard 3: Students will learn and apply geometric concepts.

Objectives:

- Use angle congruence properties
- Prove properties about special pairs of angles.

Theorem 2.2 Properties of Angle Congruence

- Angle congruence is reflexive, symmetric, and transitive.
- Examples:
  - Reflexive: For any angle A, ∠A ≅ ∠A.
  - Symmetric: If ∠A ≅ ∠B, then ∠B ≅ ∠A
  - Transitive: If ∠A ≅ ∠B and ∠B ≅ ∠C, then ∠A ≅ ∠C.

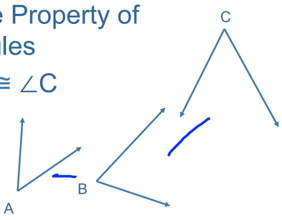
Ex. 1: Transitive Property of Angle Congruence

- Prove the Transitive Property of Congruence for angles

Given: ∠A ≅ ∠B, ∠B ≅ ∠C

Prove: ∠A ≅ ∠C

$m\angle A = m\angle B$
$m\angle B = m\angle C$
$m\angle A = m\angle C$
$\angle A \cong \angle C$



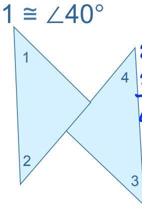
Ex. 1: Transitive Property of Angle Congruence

Statement:	Reason:
1. $\angle A \cong \angle B$, $\angle B \cong \angle C$	1. Given
2. $m\angle A = m\angle B$	2. Def. Cong. Angles
3. $m\angle B = m\angle C$	3. Def. Cong. Angles
4. $m\angle A = m\angle C$	4. Transitive property
5. $\angle B \cong \angle C$	5. Def. Cong. Angles

Ex. 2: Using the Transitive Property

Given: $m\angle 3 \cong 40°$, $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$ ←

Prove: $m\angle 1 \cong \angle 40°$



1 $\angle 1 \cong \angle 2$ $\angle 2 \cong \angle 3$ Given
2 $\angle 1 \cong \angle 3$ Transitive
3 $m\angle 1 = m\angle 3$ def of $\cong$
4 $m\angle 3 = 40°$ Given
5 $m\angle 1 = 40°$ Transitive Substitution

Ex. 2:

Statement:	Reason:
1. $m\angle 3 \cong 40°$, $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$	1. Given
2. $\angle 1 \cong \angle 3$	2. Trans. Prop of Cong.
3. $m\angle 1 \cong m\angle 3$	3. Def. Cong. Angles
4. $m\angle 1 \cong 40°$	4. Substitution

Theorem 2.3

All right angles are congruent.

Example 3: Proving Theorem 2.3

Given: $\angle 1$ and $\angle 2$ are right angles

Prove: $\angle 1 \cong \angle 2$

1. $\angle 1, \angle 2$ rt $\angle$'s
2. $m\angle 1 = 90$ def of rt $\angle$
3. $m\angle 2 = 90$ def of rt $\angle$
4. $m\angle 1 = m\angle 2$ substitution
5. $\angle 1 \cong \angle 2$ def of $\cong$

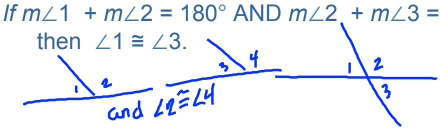
Ex. 3:

Statement:	Reason:
1. ∠1 and ∠2 are right angles	1. Given
2. $m\angle 1 = 90°$, $m\angle 2 = 90°$	2. Def. Right angle
3. $m\angle 1 \cong m\angle 2$	3. Transitive property
4. $\angle 1 \cong \angle 2$	4. Def. Cong. Angles

Properties of Special Pairs of Angles

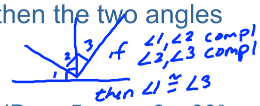
- Theorem 2.4: Congruent Supplements. If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

If $m\angle 1 + m\angle 2 = 180°$ AND $m\angle 2 + m\angle 3 = 180°$, then $\angle 1 \cong \angle 3$.



Congruent Complements Theorem

- Theorem 2.5: If two angles are complementary to the same angle (or congruent angles), then the two angles are congruent.

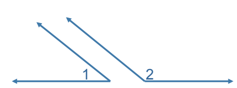


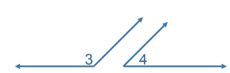
If $m\angle 4 + m\angle 5 = 90°$ AND $m\angle 5 + m\angle 6 = 90°$, then $\angle 4 \cong \angle 6$.

Proving Theorem 2.4

Given: ∠1 and ∠2 are supplements, ∠3 and ∠4 are supplements, ∠1 ≅ ∠4

Prove: ∠2 ≅ ∠3



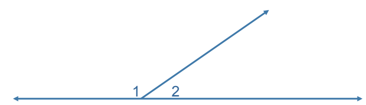


Ex. 4:

Statement:	Reason:
1. $\angle 1$ and $\angle 2$ are supplements, $\angle 3$ and $\angle 4$ are supplements, $\angle 1 \cong \angle 4$	1. Given
2. $m\angle 1 + m\angle 2 = 180°$; $m\angle 3 + m\angle 4 = 180°$	2. Def. Supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	3. Transitive property of equality
4. $m\angle 1 = m\angle 4$	2. Def. Congruent Angles
5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	3. Substitution property
6. $m\angle 2 = m\angle 3$	4. Subtraction property
7. $\angle 2 \cong \angle 3$	5. Def. Congruent Angles

Postulate 12: Linear Pair Postulate

If two angles form a linear pair, then they are supplementary.



$m\angle 1 + m\angle 2 = 180°$

Example 5: Using Linear Pairs

In the diagram, $m\angle 8 = m\angle 5$ and $m\angle 5 = 125°$.

Explain how to show $m\angle 7 = 55°$

$m\angle 7 + 125° = 180$

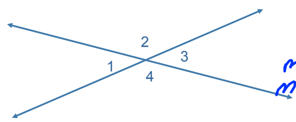
$m\angle 7 = 55$

Solution:

- Using the transitive property of equality $m\angle 8 = 125°$. The diagram shows that $m\angle 7 + m\angle 8 = 180°$. Substitute 125° for $m\angle 8$ to show $m\angle 7 = 55°$.

Vertical Angles Theorem

- Vertical angles are congruent.



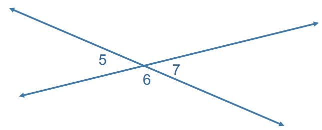
$m\angle 2 + m\angle 3 = 180$
$m\angle 3 + m\angle 4 = 180$
$m\angle 2 = 180 - m\angle 3$
$m\angle 4 = 180 - m\angle 3$
$m\angle 2 = m\angle 4$

∠1 ≅ ∠3; ∠2 ≅ ∠4 $\angle 2 \cong \angle 4$

Proving Theorem 2.6

Given: ∠5 and ∠6 are a linear pair, ∠6 and ∠7 are a linear pair

Prove: ∠5 ≅ ∠7



Ex. 6: Proving Theorem 2.6

Statement:	Reason:
1. *∠5 and ∠6 are a linear pair, ∠6 and ∠7 are a linear pair*	1. Given
2. *∠5 and ∠6 are supplementary, ∠6 and ∠7 are supplementary*	2. Linear Pair postulate
3. *∠5 ≅ ∠7*	3. Congruent Supplements Theorem