2.3 Deductive Reasoning

Rules Properties
Definitions

Standards/Objectives

Objectives:

- Use symbolic notation to represent logical statements.
- Form conclusions by applying the laws of logic to true statements, such as statements about a trip to Alabama.

2 laws

Conditional Statement

- If the sun is out (p), then the weather is good(q). → = implies
- Symbolically written as
  If p, then q or $p \rightarrow q$
- Converse, simply switch p and q.
  If q, then p or $q \rightarrow p$

Biconditional Statement

- Written symbolically:
  If p, then q and if q, then p. or
  $p \Leftrightarrow q$ $p \leftrightarrow q$
  Most often written in this form:
  p if and only if q.

Example 1: Using Symbolic Notation

- Let p be "the value of x is -5" and let q be "the absolute value of x is 5."
  - A. Write p → q in words.
  - B. Write q → p in words.
  - C. Decide whether the biconditional statement p ⇔q is true.

Solution

- A. If the value of x is -5, then the absolute value of x is 5.
- B. If the absolute value of x is 5, then the value of x is -5.
- C. The conditional statement in part a is true, but its converse (b) is false. So, the biconditional p ⇔ q is false.

Symbol for negation

- When writing negation, use the ~ symbol.
- ∠3 measures 90° -- p
- ∠3 is not acute – q
- Negation -- ∠3 does not measure 90° -- ~p
- Negation-- ∠3 is acute -- ~q

Example 2: Writing an inverse and contrapositive

- Let p be "it is raining" and let q be "the soccer game is cancelled."
  - Write the contrapositive of p → q change order negate
  - Write the inverse of p → q negate
- Answers
  - ~q → ~p—If the soccer game is not canceled, then it is not raining.
  - ~p → ~q—If it is not raining, then the soccer game is not canceled.

REMEMBER

- A CONDITIONAL STATEMENT is equivalent to its contrapositive and that the converse and inverse are equivalent.

Equivalent Statements:

- Conditional statement
  - $p \rightarrow q$ – If the car will start, then the battery is charged.
- Contrapositive
  - $\sim q \rightarrow \sim p$ – If the battery is not charged, then the car will not start.

Equivalent Statements

- Converse
  - $q \rightarrow p$ – If the battery is charged, then the car will start.
- Inverse
  - $\sim p \rightarrow \sim q$ –If the car will not start, then the battery is not charged.

Using the Laws of Logic

- Definition:
  - Deductive reasoning uses facts, definitions, and accepted properties in a logical order to write a logical argument. This differs from inductive reasoning, in whch previous examples and patterns are used to form a conjecture.

Example 3: Using inductive reasoning

- Andrea knows that Robin is a sophomore and Todd is a junior. All the other juniors that Andrea knows are older than Robin. Therefore, Andrea reasons inductively that Todd is older than Robin based on past observations.

Deductive Reasoning

- Andrea knows that Todd is older than Chan. She also knows that Chan is older than Robin. Andrea reasons deductively that Todd is older than Robin based on accepted statements.

Law of Detachment

- If $p \rightarrow q$ is a true conditional statement and p is true, then q is true.
- Example: If two angles form a linear pair, then they are supplementary; $\angle A$ and $\angle B$ are supplementary. So, $\angle A$ and $\angle B$ form a linear pair.
  - What about two separate angles whose sums happen to add up to 180^{o} but aren't adjacent.

Law of syllogism

- If $p \rightarrow q$ and $q \rightarrow r$ are true conditional statements, then $p \rightarrow r$ is true.
- Example 5: Using the law of syllogism
  - If a bird is the fastest bird on land, then it is the largest of all birds.
  - If a bird is the largest of all birds, then it is an ostrich.
  - If a bird is a bee hummingbird, then it is the smallest of all birds.
  - If a bird is the largest of all birds, then it is flightless.
  - If a bird is the smallest bird, then it has a nest the size of a walnut half-shell.

If I buy Pizza, I eat ~~as~~ much
If I eat too much Then I go to bed

Example 5 continued

- A. If a bird is the fastest bird on land, then it is an ostrich. (use 1 and 2.)
- B. If a bird is a hummingbird, then it has a nest the size of a walnut half-shell (Use 3 and 5).
- C. If a bird is the fastest bird on land, then it is flightless (Use 1 and 4).

Example: Deductive Reasoning

- If Mike visits Alabama, then he will spend a day in Montgomery.
- If Mike spends a day in Montgomery, then he will visit the Civil Rights Memorial.

Solution

- Let p, q and r represent the following:
  - P: Mike visits Alabama
  - Q: Mike spends a day in Montgomery.
  - R: Mike visits the Civil Rights Memorial
  - $p \rightarrow q$ is true
  - $q \rightarrow r$ is true; so
  - $p \rightarrow r$ is true (Law of Syllogism).

In other words

- If Mike visits Alabama, then he will visit the Civil Rights Memorial.
- You are told that Mike visited Alabama which means p is true. Using the Law of Detachment, you can conclude that he visited the Civil Rights Memorial.